Congestion control protocol for connection oriented networks
with aperiodic feedback and non-persistent sources

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Abstract: In this paper the congestion control problem in a connection-oriented communication network is addressed. In the considered multi-source network the feedback information is provided by means of control units generated by each source once every $M$ data packets. Since the sources adjust the transmission rate only at the control unit arrival, the interval between successive rate modifications is input dependent and varies with time. A new, nonlinear strategy effectively combining the Smith principle with the proportional controller with saturation is proposed. Conditions for data loss elimination and full bottleneck link bandwidth utilization are formulated and strictly proved. The presented strategy allows for full resource usage even though the sources are not persistent. Moreover, since the controller saturation limits are fully adjustable, the algorithm performance may be easily tuned according to the existing system resources.

Keywords: congestion control, connection-oriented networks, sampled data systems

1. Introduction

In recent years, the rapid expansion of long distance network traffic has triggered intensive research aimed at improving efficiency of data transmission [1–15]. A valuable survey of earlier congestion control mechanisms is given in [8]. Izmailov [6] considered a single connection controlled by a linear regulator whose output signal is generated according to the several states of the buffer measured at different time instants. The asymptotic stability, nonoscillatory system behavior and locally optimal rate of convergence have been proved. Chong et al. proposed and thoroughly studied the performance
of a simple queue length based flow control algorithm with a dynamic queue threshold adjustment [3]. Lengliz and Kamoun [10] introduced a proportional plus derivative controller which is computationally efficient and can be easily implemented in connection-oriented networks. Adaptive control strategies for data flow regulation have been proposed by Laberteaux et al. [9]. Their strategies reduce convergence time and improve queue length management. Imer et al. [5] gave a brief, excellent tutorial exposition of the congestion control problem and presented new stochastic and deterministic control algorithms. Another interesting approach to the problem of the flow rate control in communication networks has been proposed by Quet et al. in [14]. They considered a single bottleneck multi-source network and applied minimization of an H-infinity norm to the design of a flow rate controller. The proposed controller guarantees stability robustness to uncertain and time-varying propagation delays in various channels. The flow control problem in the presence of uncertain, time-varying delays was also considered by Sichitiu and Bauer [15]. Additionally, the authors of [15] demonstrated that for congestion control systems with linear controllers the stability of the system with one source is equivalent to the stability of the system with multiple sources. Also, a neural network controller for wide area networks has recently been proposed. Jagannathan and Talluri [7] showed that their neural network controller can guarantee stability of the closed loop system and the desired QoS.

Due to significant propagation delays which are critical for the closed loop performance, several researchers applied the Smith principle [16] to control the flow of data in communication networks [1, 2, 4, 11, 12]. In [11], Mascolo considered a single connection congestion control problem in a general packet switching network. He used the deterministic fluid model approximation of packet flow and applied transfer functions to describe the network dynamics. In the next paper [12], the same author applied the Smith principle to the network supporting multiple connections with different propagation delays. The proposed control algorithm guarantees no data loss, full network utilization, and ensures exponential convergence of queue levels to stationary values without oscillations or overshoots. Gómez-Stern et al. further studied the flow control using the Smith principle [4]. They proposed a continuous time proportional-integral (PI) controller which helps reduce the average queue level and its sensitivity to the available bandwidth. On the other hand, in paper [1] linear, discrete-time flow control strategy for connection-oriented networks has been proposed. The strategy combines the Smith principle with the discrete time proportional controller. A nonlinear algorithm exploiting the idea of the Smith prediction for the flow regulation was suggested in [2]. The described continuous time control mechanism guarantees congestion alleviating features and full resource usage even though the propagation delays in the multi-source network can be determined only with limited degree of accuracy.
In this paper, the flow control in a connection-oriented communication network is considered. Our approach is similar to that introduced in [1, 2, 4, 11, 12], however as opposed to [1, 4, 11, 12], we propose a nonlinear control strategy. Moreover, in contrast to [2, 4, 11, 12], where continuous time control schemes were elaborated and [1], where discrete-time controller with constant sampling period was proposed, in this paper, an algorithm, which explicitly takes into account irregularities in the feedback information availability, is presented. The strategy, combining the Smith principle with the proportional controller with saturation, guarantees full bottleneck node link utilization and no data loss in the considered model of a connection-oriented network. As a result, the need of packet retransmission is eliminated and the maximum throughput is achieved. The main novelty of the proposed strategy, as compared with the earlier results based on the Smith’s principle, is that it allows for entire bandwidth usage at the bottleneck link even though the sources do not always obey the rate adjustment command (due to temporal or inherent transfer limitations) and may deliver less data than determined by the controller.

The remainder of this paper is organized as follows. The model of the network used throughout the paper is described in Section 2. The description focuses on the properties of system with time-varying sampling period and non-persistent sources. In Section 3, the nonlinear flow control strategy is introduced, and its features are analyzed and strictly proved. Afterwards, the discussed properties are verified by a simulation example presented in Section 4. Concluding remarks are given in Section 5.

2. Network model

The telecommunication network considered in this paper consists of data sources, intermediate nodes connected via bidirectional links and destinations. The data from each source traverses a number of switches, which operate in the store-and-forward mode without the traffic prioritization, to be finally delivered to its destination. However, somewhere on the transmission path a node is encountered, whose output link cannot handle the incoming flow. Consequently, congestion occurs and packets, which constitute the data stream, accumulate in the buffer allocated for that link.

In a general case, \( n \) data flows pass through the bottleneck node and its output connection and, hence, participate in the flow regulation process. The feedback mechanism for the input rate adaptation is provided by means of control units sent by each source every \( M \) data packets. These special units travel along the same path as data packets. However, unlike data packets, they are not stored in the queues at the intermediate nodes. Instead, once they appear at the switch input link and the feedback information is incorporated, they are immediately transferred at the appropriate output port. As soon as control units reach destination, they are turned back to be retrieved at the origin and to
be used for the transfer speed adjustment round trip time after they were generated. The input rate adaptation at the instant of the management unit arrival only is justified by the fact that this is a specific moment, when the meaningful information about the current system condition is delivered to a source. Since control units are generated every $M$ ordinary packets, the time period between the arrivals of the consecutive management units depends on the emission rate round trip time earlier, which, in turn, changes according to the variations of the network state and makes the feedback unisochronic.

The presented scenario is illustrated in Fig. 1. Source $j$ ($j = 1, 2, ..., n$) sends data at the rate $a_j(t)$, where $t$ denotes time. After forward propagation delay $T_{fj}$ packets belonging to the $j$-th virtual connection reach the bottleneck node and are served according to the bandwidth availability at the output link. The remaining data accumulates in the queue. The controller placed at the bottleneck switch compares the current queue length, which at time $t$ will be denoted as $x(t)$, with its demand value $x_d$, and calculates the aggregate transmission speed $\tilde{a}(t)$. The $n$-th share of the total rate is recorded as the feedback information in every management unit passing through the node. Once the control units from source $j$ appear at the respective end system, they are turned back to arrive at their origin backward propagation delay $T_{bj}$ after being processed by the switch. Sampling module (SM) extracts the flow rate from control units and adjusts the transfer speed of the source. Since control units are not subject to queuing delays, round trip time $RTT_j = T_{fj} + T_{bj}$ remains constant for the duration of the connection for each flow.

The available bandwidth is modeled as an a priori unknown, bounded function of time $d(t)$. It is lower-bounded by a positive real constant $d_{\min}$ and limited from above by the maximum value $d_{\max}$, i.e.
Notice that this definition of the available bandwidth is quite general and it accounts for any standard distribution typically analyzed in the considered problem.

If there are packets ready for transmission in the buffer, then the bandwidth actually consumed by all the sources $h(t)$ will be equal to the available bandwidth. Otherwise, the output link is underutilized and the exploited bandwidth matches the data arrival rate at the node. Thus, we may write

$$0 \leq h(t) \leq d(t) \leq d_{\text{max}}. \quad (2)$$

Let us denote by $b_j(t)$ the rate calculated by the controller and sent for the $j$-th source at the instant of a management unit passing through the node. Once the control unit arrives back at the source, the input rate is adjusted to the value determined by the bottleneck node. However, the source cannot always obey the command due to inherent or temporal transfer limitations. Assuming that the sources begin transmission at the time $t = 0$, the $j$-th source rate $a_j(t)$ can be expressed as follows

$$\forall \forall \ a_j(t) = 0 \quad \text{and} \quad \forall \forall \ a_j(t) = f_j(t)b_j(t - T_{fj}), \quad (3)$$

where $f_j(t)$, representing the source transfer limitations, is subject to the constraint

$$0 < f_{\min} \leq f_j(t) \leq 1. \quad (4)$$

Since signal $b_j(t)$ constitutes a vital part of the proposed control scheme, its proper definition will be given together with the description of the flow regulation strategy in the subsequent section.

The rate of change of the queue length at any instant of time depends on the data arrival speed and on the buffer depletion rate, which, in turn, is directly related to the consumed bandwidth $h(t)$. As a consequence of (3) no packet arrives at the congested node before $T_{f_{\text{min}}} = \min_{j=1,2,...,n} (T_{fj})$. Assuming that initially (before the time $t = 0$) the bottleneck buffer was empty, i.e. $x(t < 0) = 0$, the queue length remains at the zero level for any time instant smaller than or equal to $T_{f_{\text{min}}}$, i.e. $x(t \leq T_{f_{\text{min}}}) = 0$. Afterwards, for any $t > T_{f_{\text{min}}}$ the length of the queue at the node may be expressed as

$$x(t) = \sum_{j=1}^{n} \int_{T_{fj}}^{t} a_j(\tau - T_{fj})d\tau - \int_{0}^{t} h(\tau)d\tau. \quad (5)$$

Let us denote by $t_{jk}$ the $k$-th moment of time ($k = 1, 2, ...$) when the control unit belonging to the $j$-th virtual connection data flow arrives back at source $j$. We assume
that the first packet transferred by each source is a control unit so that the information about the current network state could be received at the data origin as quickly as possible. As the sources begin transmission at the time instant \( t = 0 \), then for \( k = 1 \) we have \( t_{j,1} = RTT_j \). Furthermore, since control units are sent every \( M \) data packets, \( t_{j,k+1} \) can be determined from the following relation

\[
\int_{t_{j,k}}^{t_{j,k+1}} a_j(\tau - RTT_j) d\tau = M.
\]

Definition (6) makes sense only for the nonnegative rates \( a_j(t) \). Clearly, any control algorithm should be constructed in such a way that this condition is satisfied for every signal \( a_j(t) \). Moreover, although the sources cannot always transmit data at the rate established by the controller, we assume that \( a_j(t) \) cannot be lower than the minimum rate \( a_{\text{min}}/n \) in order to provide basic responsiveness to the changes of networking conditions. Hence, each source emits a control unit (and after \( RTT \) adjusts the transfer speed) at least every \( T_C = Mn/a_{\text{min}} \).

The presented model reflects the behavior of connection-oriented communication networks where the feedback information is delivered with priority over the data traffic. A good example of such systems is ATM (Asynchronous Transfer Mode) technology with feedback distributed in RM (Resource Management) cells emitted every \( M \) (usually 32) data cells.

### 3. Control algorithm

In this section we formulate a new control law for the considered multi-source network with aperiodic feedback information distribution. We begin with stating the design goals. Afterwards, the algorithm principle is presented and its properties discussed, and proved mathematically.

Modern telecommunication systems demand efficient resource exploitation. Therefore, a successful control strategy should guarantee that

(i) data is not lost due to congestion,

(ii) and there are always some data packets ready for the transmission in the buffer so that the available bandwidth at the output link of the bottleneck switch is entirely utilized.

In the sequel we propose a new, nonlinear flow regulation algorithm, which fulfills these goals.
The rate calculated by the controller $\bar{a}(t)$ is determined from the relation given below

$$
\bar{a}(t) = \begin{cases} 
    a_{\text{min}}, & \text{if } W(t) < a_{\text{min}} \\
    W(t), & \text{if } a_{\text{min}} \leq W(t) \leq a_{\text{max}} \\
    a_{\text{max}}, & \text{if } W(t) > a_{\text{max}}
\end{cases}
$$

(7)

where $a_{\text{min}}$ and $a_{\text{max}}$ denote the lower and upper limits of the possible flow rate values and $0 < a_{\text{min}} < a_{\text{max}}$. We define function $W(t)$ as

$$
W(t) = K [x_d - x(t) - S(t)],
$$

(8)

where $K > 0$ is the controller gain and $x_d > 0$ is the demand queue length. The component

$$
S(t) = \sum_{j=1}^{n} \int_{t-RTT_j}^{t} b_j(\tau)d\tau
$$

(9)

is responsible for the Smith prediction and it represents the ‘in-flight’ data. The rate $b_j(t)$, sent and recorded by the switch for source $j$ at the instant of a control unit passing through the node, is determined from the following equation

$$
b_j(t) = \begin{cases} 
    0, & \text{for } t < -T_{b_j} \\
    a_{\text{min}}/n, & \text{for } -T_{b_j} \leq t < T_{fj} \\
    \bar{a}(t_{jk} - T_{b_j})/n, & \text{for } t \geq T_{fj} \text{ and } t \in [t_{jk} - T_{b_j}, t_{jk+1} - T_{b_j}]
\end{cases}
$$

(10)

Therefore, source $j$ sends data to the network at the minimum rate $a_{\text{min}}/n$ in the $[0, RTT_j]$ interval. Afterwards, it delivers packets at the rate determined by the controller (according to (8) and (10)) at discrete time instants $t_{jk} - T_{b}$.

We assume that in order for the network to be able to transport the user data at least at the minimum rate, $a_{\text{min}} \leq d_{\text{min}}$. On the other hand, to make it possible to always exploit all of the available bandwidth, we expect $a_{\text{max}} \geq d_{\text{max}}$. Therefore, since the utilized bandwidth is limited from below by either the available bandwidth or the incoming flow rate (when the queue length is zero), $h(t)$ will satisfy the following inequalities

$$
a_{\text{min}} \leq h(t) \leq d_{\text{max}} \leq a_{\text{max}}.
$$

(11)

To fulfill the first design objective (i), it suffices to show that the queue length never increases beyond a given limit irrespective of the bandwidth changes. Then, if we assign the buffer capacity at least equal to the indicated maximum, no packet will be discarded as a result of congestion, and the risk of losing data will be eliminated.
Theorem 1: If sources transmit data according to the conditions formulated by (3)–(10), then for any $t \geq 0$ the queue length at the bottleneck node is upper-bounded by $q_{\text{max}}$, where

$$q_{\text{max}} = x_d - a_{\text{min}} (\lambda + 1/K) + (a_{\text{max}} - a_{\text{min}}) T_C,$$

(12)

provided that

$$x_d > a_{\text{max}} (\lambda + 1/K),$$

(13)

where $\lambda = \sum_{j=1}^{n} \text{RTT}_j/n$ denotes the mean round trip time for the considered set of $n$ connections.

**Proof:** The rate is adjusted by each source at discrete time instants $t_{j,k}$, and the effect of the modification affects the total arrival rate at the bottleneck node after forward propagation delay. Let us denote by $\theta_m (m = 1, 2, ...)$ the moment of time when the total arrival rate changes as a consequence of the transfer speed adjustment at some source. The first such modification coincides with the arrival of the initial packet belonging to the flow with the shortest forward delay, so for $m = 1$ we have $\theta_1 = T_{f_{\text{min}}}$. Since each source updates the transmission rate at least every $T_C$, the interval

$$\alpha_m = \theta_{m+1} - \theta_m$$

(14)

between any two consecutive potential changes of the total incoming rate at the congested switch is subject to the constraint $0 \leq \alpha_m \leq T_C$. The interval $\alpha_m = 0$ reflects the case, when the modification of the transmission speed, which occurred at two or more sources, influences the aggregate rate at the switch at the same moment of time.

Let us denote the queue length at time instant $\theta_m$ by $x_m = x(\theta_m)$. For $m = 1$ and $\theta_1 = T_{f_{\text{min}}}$ we can write $x_1 = x(\theta_1) = 0 < q_{\text{max}}$. Therefore, the proposition holds for any moment of time $t \leq T_{f_{\text{min}}}$.

Let us consider some $m > 1$ and the queue length at a time instant $t \in [\theta_m, \theta_{m+1})$

$$x(t) = x_m + P(\theta_m, \theta_m + \delta) - \int_{\theta_m}^{\theta_m+\delta} h(\tau)d\tau,$$

(15)

where $t = \theta_m + \delta$ and $\delta \in [0, \alpha_m)$. The function

$$P(\tau_1, \tau_2) = \sum_{j=1}^{n} \int_{\tau_1}^{\tau_2} a_j (\tau - T_{f_j})d\tau$$

(16)

represents the amount of data that arrived at the bottleneck node between time instants $\tau_1$ and $\tau_2$. 
In order to analyze the queue length variations in time interval \((\theta_m, \theta_{m+1})\), we examine the behavior of function \(W\). We will consider two cases: first, the situation when \(W(\theta_m) > a_{\text{min}}\), and, afterwards, the circumstances when \(W(\theta_m) \leq a_{\text{min}}\).

**Case 1:** We analyze the situation when \(W(\theta_m) > a_{\text{min}}\). From the definition of function \(W\), we get

\[
x_m < x_d - \frac{a_{\text{min}}}{K} - S(\theta_m).
\]

(17)

Assumption (13) guarantees that the expression on the right-hand side of inequality (17) is positive. Applying (17) to (15), we arrive at

\[
x(t) < x_d - \frac{a_{\text{min}}}{K} - S(\theta_m) + P(\theta_m, \theta_m + \delta) - \int_{\theta_m}^{\theta_m + \delta} h(\tau)d\tau.
\]

(18)

The biggest amount of data arrives at the node if all the sources are able to follow the controller command, i.e. if \(\forall j f_j(t) = 1\). On the other hand, the quantity on the right-hand side of (18) is the biggest in the situation when \(\delta > RTT_j\). Then, the number of incoming packets exceeds the predicted one. Consequently, since the individual source rate is upper-bounded by \(a_{\text{max}}/n\), the difference \(P(\theta_m, \theta_m + \delta) - S(\theta_m)\) can be evaluated as

\[
P(\theta_m, \theta_m + \delta) - S(\theta_m) \leq a_{\text{max}} (\delta - \lambda).
\]

(19)

Using (11) and (19), we get the following estimate of the queue length (18)

\[
x(t) < x_d - \frac{a_{\text{min}}}{K} + a_{\text{max}} (\delta - \lambda) - a_{\text{min}}\delta =
\]

\[
x_d - a_{\text{min}} (\lambda + 1/K) + (a_{\text{max}} - a_{\text{min}}) (\delta - \lambda).
\]

(20)

Since \(\delta \leq T_C\),

\[
x(t) < x_d - a_{\text{min}} (\lambda + 1/K) + (a_{\text{max}} - a_{\text{min}}) (T_C - \lambda) \leq q_{\text{max}}.
\]

(21)

This ends the first part of the proof.

**Case 2:** Now, let us examine the situation when \(W(\theta_m) \leq a_{\text{min}}\). We begin with finding the last moment \(r' < \theta_m\) when signal \(W\) was greater than \(a_{\text{min}}\). It should be stressed at this point, that since the control unit emission at the sources and the rate generation at the congested node are not synchronized, \(r'\) does not have to coincide with any of the \(\theta_m\) time instants. According to (10) and (13), \(W(-T_{b_{\text{max}}})\), where \(T_{b_{\text{max}}} = \max_{j=1,2,...,n} (T_{b_j})\), satisfies the following

\[
W(-T_{b_{\text{max}}}) = K (x_d - 0 - 0) > a_{\text{max}} > a_{\text{min}}.
\]

(22)
For \( t \in (-T_{b_{\text{max}}}, T_{f_{\text{min}}}) \) signal \( W \) decreases with time and, due to assumption (13), at the upper margin of the indicated interval attains the value greater than \( a_{\text{min}} \), i.e. \( W(T_{f_{\text{min}}}) > a_{\text{min}} \). This means that moment \( t^* \) actually exists and \( t^* > T_{f_{\text{min}}} \). The value of \( W(t^*) \) satisfies the following inequality
\[
W(t^*) = K [x_d - x(t^*) - S(t^*)] > a_{\text{min}}.
\] (23)

After the term rearrangement, we obtain
\[
x(t^*) < x_d - a_{\text{min}}/K - S(t^*).
\] (24)

The queue length at a time instant \( t \in [\theta_m, \theta_{m+1}) \) can be expressed as
\[
x(t) = x(t^*) + P(t^*, t) - \int_{t^*}^{t} h(\tau) d\tau,
\] (25)
where \( t = \theta_m + \delta, \delta \in [0, \alpha_m) \) and \( \alpha_m \) is defined by (14). Applying (24) to (25), we get
\[
x(t) < x_d - a_{\text{min}}/K - S(t^*) + P(t^*, t) - \int_{t^*}^{t} h(\tau) d\tau.
\] (26)

The term \( P(t^*, t) - S(t^*) \) describes the difference between the number of packets which arrived at the bottleneck node during the interval from \( t^* \) to \( t \), and the number of packets still ‘in flight’ at the time moment \( t^* \), i.e. those for which the sending rate has already been calculated and which have not yet arrived at the buffer of the bottleneck node. The biggest number of packets will arrive if the sources have no limitations, i.e. if \( \forall j f_j(t) = 1 \). Then, \( a_j(t) = b_j(t - T_{b_j}) \), and from definitions (9) and (16) we obtain
\[
P(t^*, t) - S(t^*) = \sum_{j=1}^{n} \int_{t^*}^{t} b_j(\tau) d\tau - S(t).
\] (27)

Time instant \( t^* \) is the last moment when the algorithm computed a value greater than \( a_{\text{min}} \). Afterwards, the established flow rate was equal to \( a_{\text{min}} \), so difference (27) can be evaluated as
\[
\sum_{j=1}^{n} \int_{t^*}^{t} b_j(\tau) d\tau - S(t) \leq a_{\text{max}} T_C + a_{\text{min}} (t - t^* - T_C) - a_{\text{min}}.l.
\] (28)

Recall that \( h(t \geq T_{f_{\text{min}}}) \geq a_{\text{min}} \). Then, using (28) we get the following estimate of the queue length (26)
\[ x(t) < x_d - a_{\text{min}}/K + a_{\text{max}}T_C + a_{\text{min}}(t - t^*) - a_{\text{min}}\lambda - a_{\text{min}}(t - t^*) = \]
\[ = x_d - a_{\text{min}}(\lambda - 1/K) + (a_{\text{max}} - a_{\text{min}})T_C = q_{\text{max}}. \]  

(29)

This concludes the proof.

Full link utilization, as formulated by (ii), requires the presence of data packets in the bottleneck node buffer at any instant of time. In other words, if \( x(t) \) is greater than zero, then all of the available bandwidth at the congested link is consumed. The theorem presented below shows how the demand queue length \( x_d \) should be selected so that this condition is fulfilled.

**Theorem 2:** If sources transmit data according to the conditions formulated by (3)–(10), \( f_{\text{min}}a_{\text{max}} > d_{\text{max}} \) and the demand value of the queue length satisfies the following inequality

\[ x_d > a_{\text{max}}(\lambda + 1/K) + (f_{\text{min}}a_{\text{max}} - a_{\text{min}})T_C, \]  

then for any \( t > T_{f_{\text{min}}} + T_C + T_{\text{max}} \), where \( T_{\text{max}} = q_{\text{max}}/(f_{\text{min}}a_{\text{max}} - d_{\text{max}}) \), the queue length is strictly positive.

**Proof:** The theorem assumption implies that we deal with the time instants \( t > T_{f_{\text{min}}} + T_C + T_{\text{max}} > \theta_1 \). Considering some \( m > 1 \) and the value of signal \( W \) at the moment of the node input rate modification \( \theta_m \), we may distinguish two cases: the situation when \( W(\theta_m) < a_{\text{max}} \), and the circumstances when \( W(\theta_m) \geq a_{\text{max}} \).

**Case 1:** We consider the situation when \( W(\theta_m) < a_{\text{max}} \). Directly from the definition of function \( W \), we obtain

\[ x_m > x_d - a_{\text{max}}/K - S(\theta_m). \]  

(31)

Let us examine the queue length at some time instant \( t \in [\theta_m, \theta_{m+1}) \) as was defined in (15). Using (31), we get

\[ x(t) > x_d - a_{\text{max}}/K - S(\theta_m) + P(\theta_m, \theta_m + \delta) - \int_{\theta_m}^{\theta_m+\delta} h(\tau)d\tau. \]  

(32)

The Smith predictor term \( S(\theta_m) \leq a_{\text{max}}\lambda \). On the other hand, each source transmits packets at least at the \( a_{\text{min}}/n \) rate, so \( P(\theta_m, \theta_m + \delta) \geq a_{\text{min}}\delta \). Therefore, we can state the following

\[ x(t) > x_d - a_{\text{max}}/K - a_{\text{max}}\lambda + a_{\text{min}}\delta - \int_{\theta_m}^{\theta_m+\delta} h(\tau)d\tau. \]  

(33)

Since the utilized bandwidth is upper-bounded by \( d_{\text{max}} \), the queue length expressed by (33) will satisfy the condition given below
\[ x(t) > x_d - a_{\text{max}}/K - a_{\text{max}} \lambda + a_{\text{min}} \delta - d_{\text{max}} \delta > 0. \]  

(34)

This concludes the first part of the proof.

**Case 2:** Now, let us investigate the situation when \( W(\theta_m) \geq a_{\text{max}} \). First, notice that according to (30) the assumptions of Theorem 1 are fulfilled. We seek for the last moment \( t^* < \theta_m \) when signal \( W \) was smaller than \( a_{\text{max}} \). It comes from Theorem 1 that the queue length never exceeds \( q_{\text{max}} \). At the same time, the packet depletion rate is limited by \( d_{\text{max}} \). On the other hand, in the situation when the assigned rate equals \( a_{\text{max}} \), the incoming rate at the node (after taking into account the transfer limitations of the sources) will not be lower than \( f_{\text{min}} a_{\text{max}} \). Thus, in order to preserve the buffer space indicated in Theorem 1, the controller may continuously set the biggest rate \( a_{\text{max}} \) for the maximum period \( T_{\text{max}} = q_{\text{max}}/(f_{\text{min}} a_{\text{max}} - d_{\text{max}}) \), and instant \( t^* \) actually exists.

Since \( t^* \) was the last instant, when \( W < a_{\text{max}} \), and the time separation between any two consecutive aggregate input rate modifications does not exceed \( T_C \), \( t^* \geq \theta_m - (T_{\text{max}} + T_C) \).

The value of \( W(t^*) \) satisfies the inequality given below

\[ W(t^*) = K [x_d - x(t^*) - S(t^*)] < a_{\text{max}}. \]  

(35)

Rearranging terms in (35), we get

\[ x(t^*) > x_d - a_{\text{max}}/K - S(t^*). \]  

(36)

The queue length at some moment of time \( t \in [\theta_m, \theta_{m+1}) \) may be expressed as in (25). Applying (36) to (25), we obtain

\[ x(t) > x_d - a_{\text{max}}/K - S(t^*) + P(t^*, t) - \int_{t^*}^t h(\tau) d\tau. \]  

(37)

According to (7), \( S(t^*) \leq a_{\text{max}} \lambda \). Recall that \( t^* \) was the last instant before \( t \), when the controller calculated rate smaller than \( a_{\text{max}} \). This rate can be as low as \( a_{\text{min}} \). Afterwards, the algorithm computed the maximum value of the flow rate. Since control units appear at discrete time instants, the rate assignment can be delayed, but not more than by \( T_C \). Moreover, due to the source transfer capability limitations, the incoming rate at the node (although predicted to be at the maximum) may be as low as \( f_{\text{min}} a_{\text{max}} \). Thus, the amount of the incoming data \( P \) will satisfy the following

\[ P(t^*, t) \geq a_{\text{min}} T_C + f_{\text{min}} a_{\text{max}} (t - t^* - T_C). \]  

(38)

For any \( t \) the utilized bandwidth \( h(t) \leq d_{\text{max}} \), so substituting (38) into (37), we arrive at

\[ x(t) > x_d - a_{\text{max}}/K - a_{\text{max}} \lambda + a_{\text{min}} T_C + f_{\text{min}} a_{\text{max}} (t - t^* - T_C) - d_{\text{max}} (t - t^*). \]  

(39)
The theorem assumption implies \( f_{\min} a_{\max} > d_{\max} \). Since \( t > t^* \),

\[
x(t) > x_d - a_{\max} (\lambda + 1/K) - (f_{\min} a_{\max} - a_{\min}) T_C > 0.
\]

This ends the proof.

4. Simulation results

In order to verify the control strategy proposed in this paper, simulation tests were performed using Matlab-Simulink® and discrete event simulator ns2. Since the obtained plots were very similar (differences remained in the range of numerical errors), we present only the Simulink graphs. The model of a wide area network with irregular period of feedback information availability was constructed according to the description provided in Section 2. Three connections \((n = 3)\), characterized by the delays: \( RTT_1 = 20 \text{ ms} \), \( RTT_2 = 30 \text{ ms} \) and \( RTT_3 = 70 \text{ ms} \), participate in the flow regulation process. Each source sends control units every \( M = 32 \) equal-size data pieces. The maximum available bandwidth \( d_{\max} \) was set as 9100 packets/s, the lower bound of the overall source rate \( a_{\min} \) as 960 packets/s, and the upper bound \( a_{\max} \) as 18300 > \( d_{\max}/f_{\min} = 9100/0.5 \) packets/s. The controller gain \( K \) was adjusted to 20 s\(^{-1}\) and the demand queue length \( x_d \), calculated according to (30), was set as 2470 > 2466 packets. The bandwidth actually available for the controlled connections is illustrated in Fig. 2. We can see from the graph that function \( d \) experiences sudden changes of large amplitude, which reflects the most rigorous networking conditions.

![Fig. 2. Available bandwidth](image)

The queue length \( x(t) \) resulting from applying the proposed regulation scheme is shown in Fig. 3 and the rate determined by the controller in Fig. 4. As we can see, the
queue length never exceeds the value of 4115 packets and does not drop to 0. These two properties imply no buffer overflow and full bottleneck link utilization. The rates assigned for the sources (curve a) and the true transfer speeds of each transmitter (curve b) are shown in Fig. 5. It is clear from the plots that the sources cannot always deliver data at the rate established by the controller (the actual rate can be as low as half of the assigned one). Despite significant rate limitations experienced by the transmitters, the proposed strategy guarantees the maximum throughput in the network.

Fig. 3. Queue length in the bottleneck node buffer

Fig. 4. Rate generated by the controller
In this paper the congestion control problem in a connection-oriented network supporting multiple data flows was addressed. The proposed nonlinear algorithm effectively combines the Smith principle with the proportional controller with saturation. Conditions for no packet loss and full network utilization were presented and strictly proved with explicit consideration of the irregularities in the feedback information availability. As the transfer speed limits are fully adjustable within the bandwidth constraints, the algorithm performance may be easily tuned by the user. Moreover, since the rates generated by the controller are always nonnegative and bounded, the proposed strategy may be directly implemented in real systems. The main advantage of the algorithm proposed
in this paper over previously published Smith’s predictor based solutions is that it allows for the total resource usage despite possible transfer limitations of the sources. Since low rates may result from the transmitter itself, or may be caused by congestion occurring elsewhere in the network (and not at the node where the controller operates) the proposed algorithm may efficiently coexist with other, possibly different flow regulation schemes in the communication system. In order to satisfy the max-min fairness criteria in such complex, multi-bottleneck systems an appropriate source saturation compensator, e.g. similar to the one recently proposed in [13], can be applied with the appropriately modified rate allocation procedure. In fact, this is the subject of our current research in the field of network congestion control.

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References

Protokół kontroli przeciżeń dla połączeniowych sieci teleinformatycznych z aperiodycznym sprzężeniem zwrotnym i nieidealnymi źródłami

Streszczenie

W artykule rozważano zagadnienie kontroli przeciżeń w połączeniowej sieci teleinformatycznej o wielu źródłach. W analizowanej sieci informacja zwrotna o bieżącym stanie układu przekazywana jest do nadajników w jednostkach sterujących generowanych przez każde źródło co M pakietów z danymi. Jednostki sterujące pokonują tę samą trasę, co pakiety danych, zbierając informację zwrotną z węzłów pośredniczących. W odbiorniku jednostki sterujące są zawracane i przesyłane z powrotem do źródła, gdzie docierają po okresie pełnego obiegu (ang. round trip time). Ponieważ każde źródło dostosowuje prędkość nadawania wyłącznie w momencie odebrania jednostki sterującej, okres dyskretyzacji zależy od poprzednich szybkości emisji danych, a zatem zmienia się w czasie.

W artykule zaproponowano nowy, nieliniowy algorytm sterowania prędkością nadawania danych, bezpośrednio uwzględniający zmienność okresu próbkiowania układu. Zaproponowana strategia wykorzystuje predyktor Smitha oraz regulator proporcjonalny z nasyceniem. Zastosowanie prezentowanego rozwiązania pozwala wyeliminować ryzyko gubienia danych, zarazem w pełni wykorzystać dostępne pasmo i uzyskać maksymalną wydajność sieci. Pełne wykorzystanie dostępnych zasobów zagwarantowane jest również w sytuacji, gdy źródło, z uwagi na czasowe ograniczenia, nie są w stanie zrealizować sygnału sterującego i nadają dane z prędkością mniejszą od ustalonej przez regulator. Prędkości generowane przez algorytm są zawsze nieujemne i ograniczone, co pozwala zastosować proponowaną strategię w warunkach działania rzeczywistej sieci telekomunikacyjnej. Co więcej zaprezentowany algorytm ułatwia obsługę administracyjną węża sieciowego umożliwiając elastyczną dopasowanie poziomów nasycenia regulatora do pojemności łącz przesyłowych. Wymienione właściwości zostały sformułowane w postaci twierdzeń i ściśle udowodnione, a następnie zweryfikowane symulacyjnie.